B PROBLEM SOLVING USING TRIGONOMETRY

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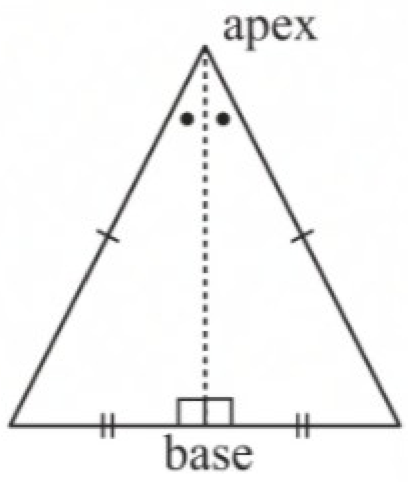
The trigonometric ratios can be used to solve a wide variety of problems involving right angled triangles. When solving these problems it is important to follow the steps below:

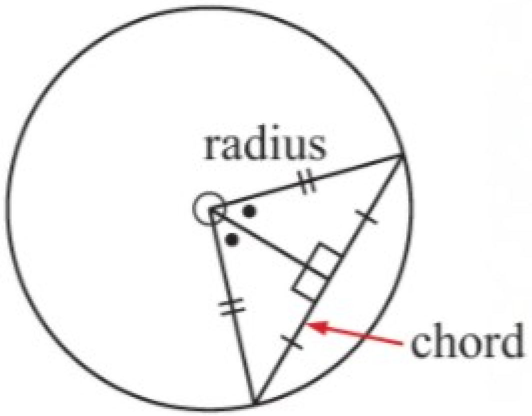
- Draw a **diagram** to illustrate the situation.
- Mark on the diagram the **unknown** angle or side that needs to be calculated. We often use x for a length and θ for an angle.
- Locate a **right angled triangle** in your diagram.
- Write an **equation** connecting an angle and two sides of the triangle using an appropriate trigonometric ratio.
- **Solve** the equation to find the unknown.
- **Write** your answer in sentence form.

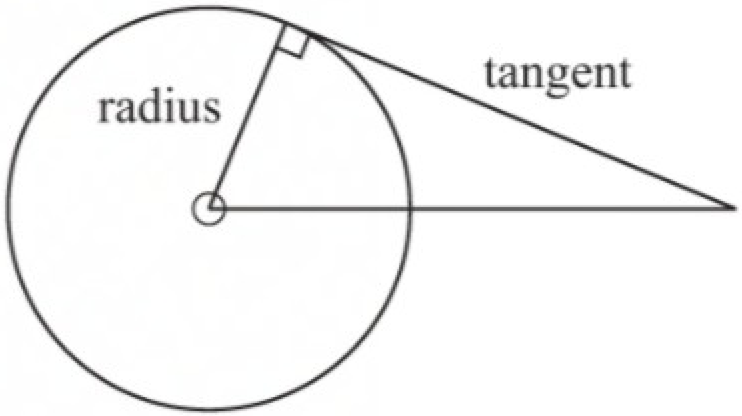
When solving problems using trigonometry, we often use:

- the properties of isosceles triangles



- the properties of circles and tangents

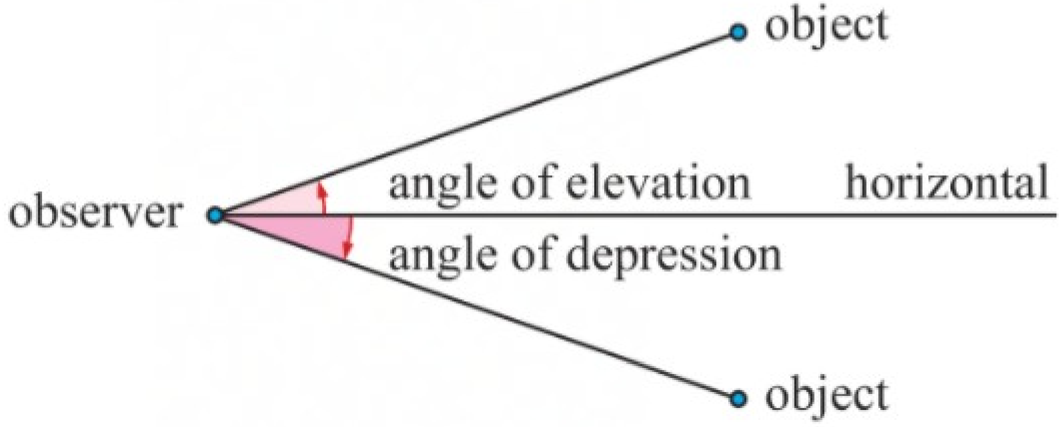




- angles of elevation and depression.

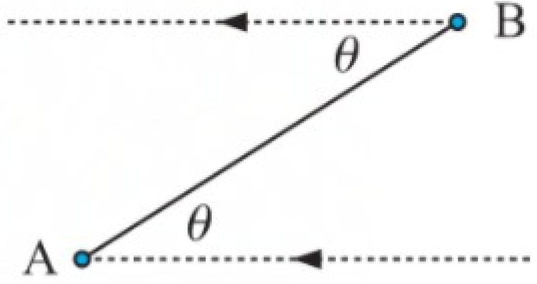
ANGLES OF ELEVATION AND DEPRESSION

When an object is **higher** than an observer, the **angle of elevation** is the angle from the horizontal **up** to the object.



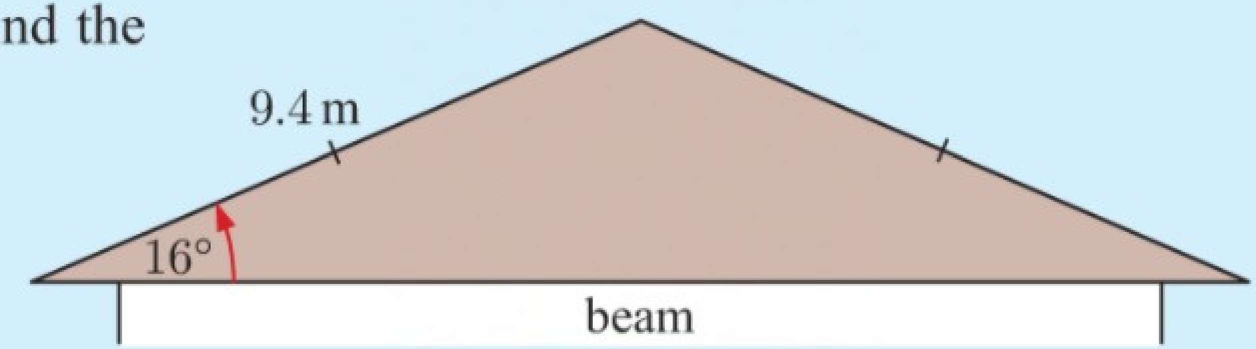
When an object is **lower** than an observer, the **angle of depression** is the angle from the horizontal **down** to the object.

If the angle of elevation from A to B is θ, then the angle of depression from B to A is also θ.



Example 3 **Self Tutor**

The roof alongside has a pitch of 16°. Find the length of the horizontal beam.



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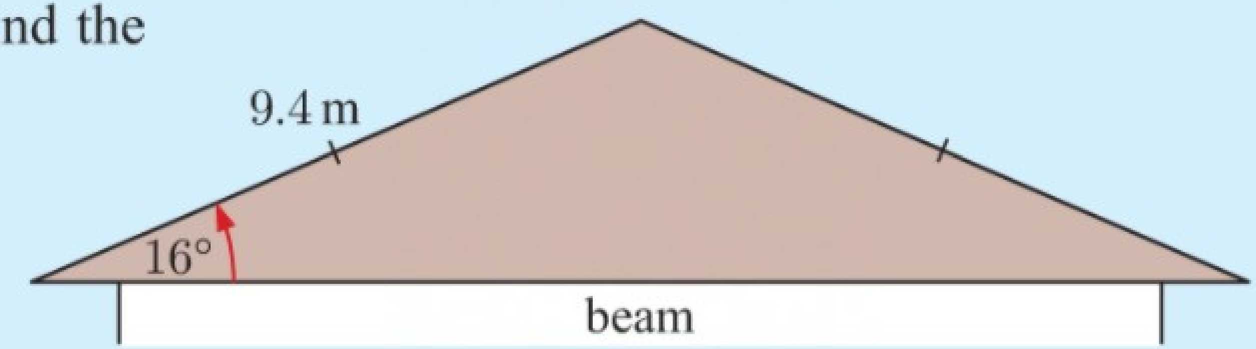
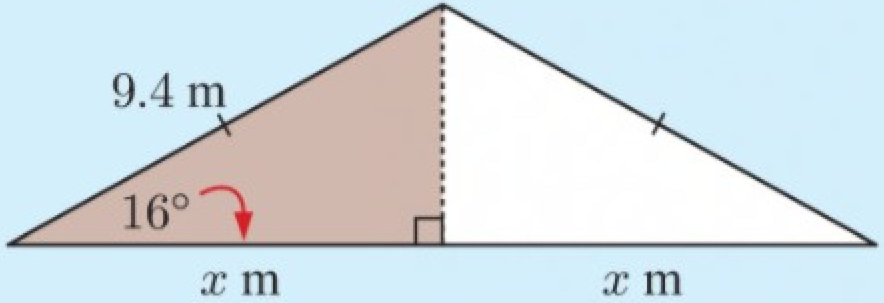




$$\cos 16° = \frac{x}{9.4} \qquad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$$

$$\therefore \quad x = 9.4 \times \cos 16°$$

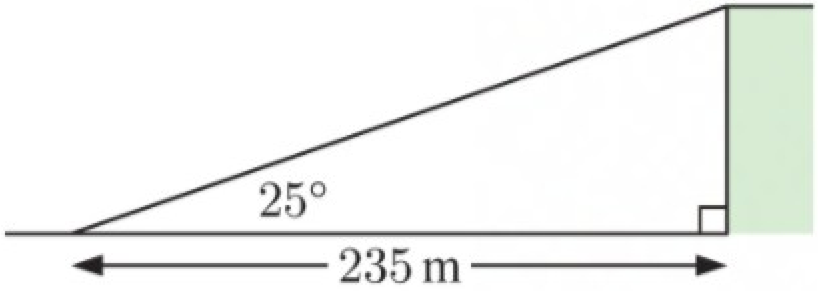
$$\therefore \quad x \approx 9.036 \qquad \{\text{calculator}\}$$

$$\therefore \quad \text{the length of the beam} \approx 2 \times 9.036 \text{ m}$$

$$\approx 18.1 \text{ m}$$

EXERCISE 12B

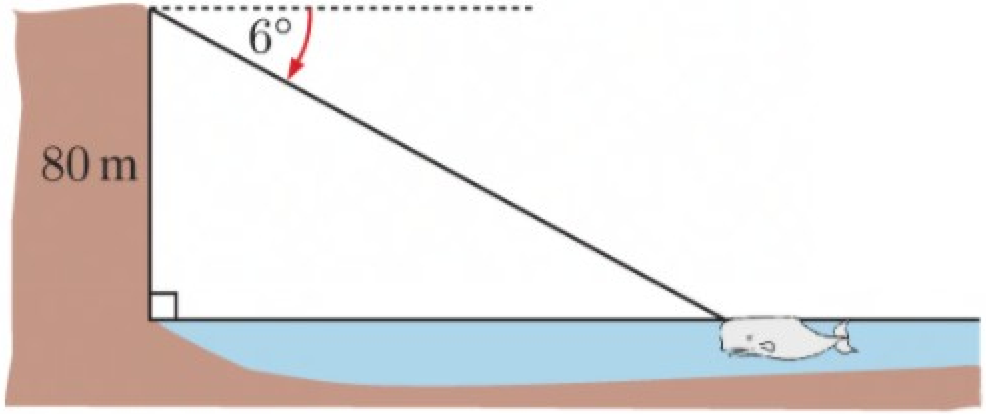
1 From a point 235 m from the base of a cliff, the angle of elevation to the cliff top is 25°. Find the height of the cliff.



2 A 5 m ladder reaches 4.2 m up a wall. What angle does the ladder make with the wall?

3 The angle of elevation from a row boat to the top of a lighthouse 25 m above sea-level is 6°. Calculate the horizontal distance from the boat to the lighthouse.

4

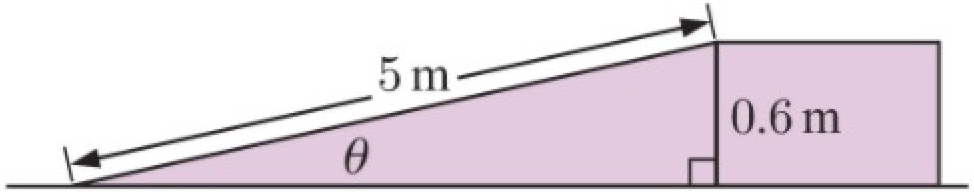


From a vertical cliff 80 m above sea level, a whale is observed at an angle of depression of 6°.

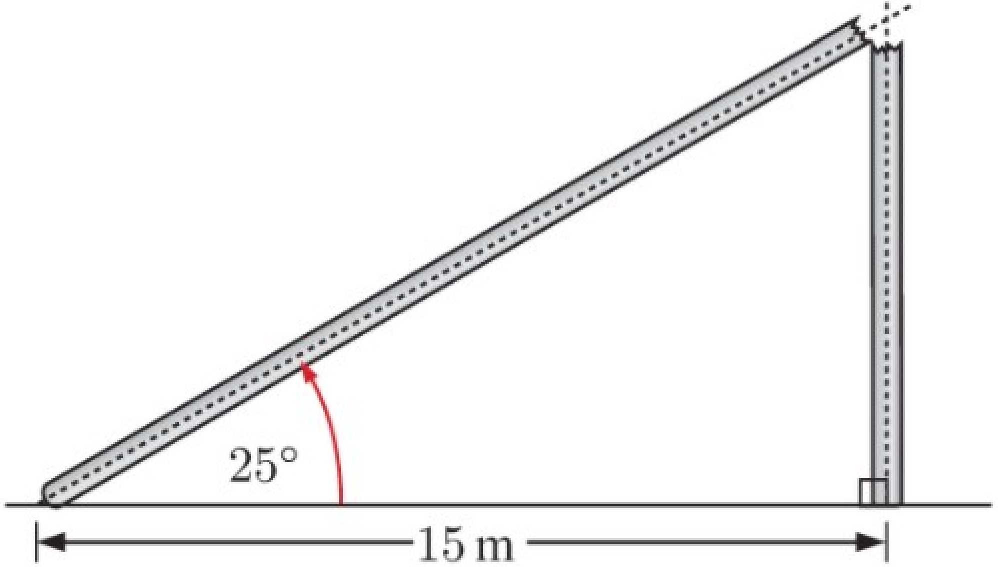
Find the distance between the observer at the top of the cliff, and the whale.

5 A train travelling up an incline of 4° travels a horizontal distance of 4 km. How much altitude has the train gained?

6 At the entrance to a building there is a ramp for wheelchair access. The length of the ramp is 5 metres, and it rises to a height of 0.6 metres. Find the angle θ that the ramp makes with the ground.



7 A goal post was hit by lightning and snapped in two. The top of the post is now resting 15 m from its base, at an angle of 25°. Find the height of the goal post before it snapped.



Example 4 **Self Tutor**

A point P is 9.5 cm from the centre C of a circle with radius 2 cm.

Find the angle between [PC] and the tangent to the circle from P.

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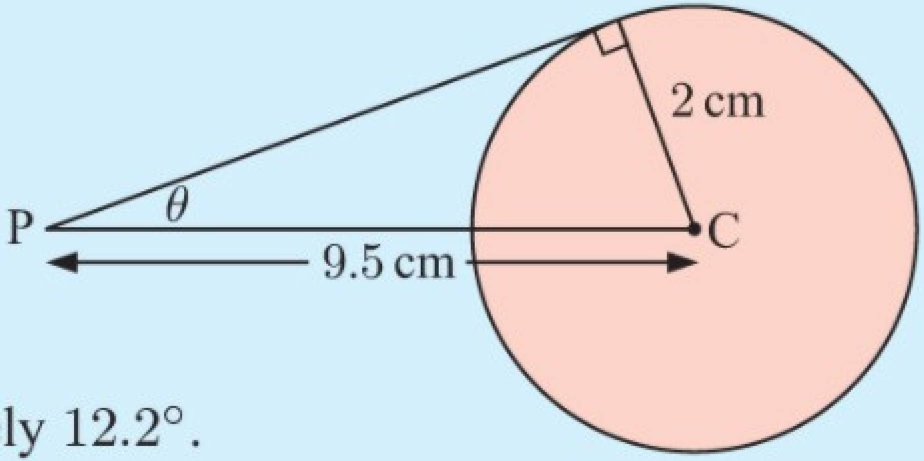


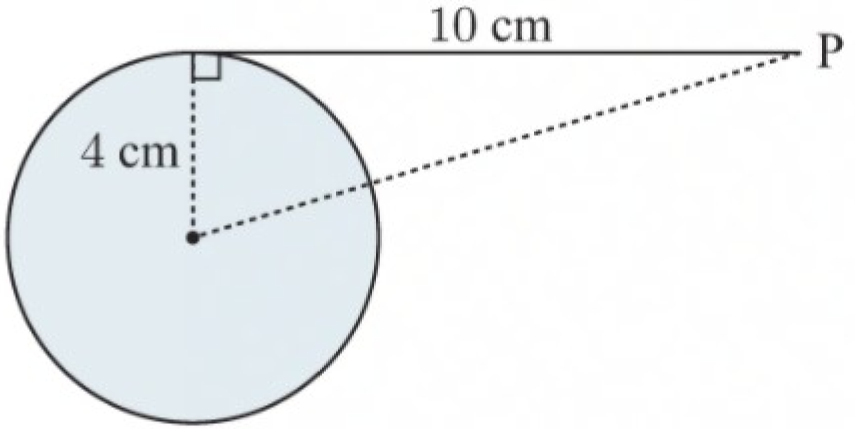
$\sin\theta = \frac{2}{9.5}$ $\quad\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \theta = \sin^{-1}\left(\frac{2}{9.5}\right)$

$\therefore\ \theta \approx 12.2^\circ$ $\quad$ {calculator}

The angle between [PC] and the tangent is approximately 12.2°.

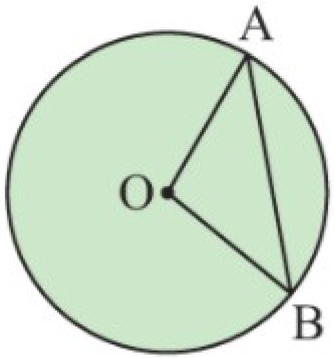
8 A tangent from point P to a circle of radius 4 cm is 10 cm long. Find the angle between the tangent and the line joining P to the centre of the circle.



9 A rhombus has sides of length 10 cm, and the angle between two adjacent sides is 76°. Find the length of the longer diagonal of the rhombus.

10 [AB] is a chord of a circle with centre O and radius 5 cm.

[AB] has length 8 cm. Find the size of $A\hat{O}B$.



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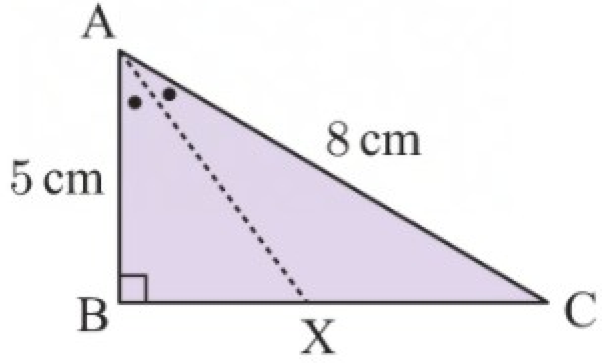


In triangle ABC, the angle bisector at A meets [BC] at X. Is X the midpoint of [BC]? If not, what is the distance between X and the midpoint?

12 In an isosceles triangle, the equal sides are $\frac{2}{3}$ of the length of the base. Determine the measure of the base angles.

13 An isosceles triangle is drawn with base angles 24° and base 28 cm. Find the base angles of the isosceles triangle with the same base length but with treble the area.

14 The angle of elevation from a marker on level ground to the top of a building 100 m high is 22°. Find the distance:

- **a** from the marker to the base of the building
- **b** the marker must be moved towards the building so that the angle of elevation becomes 40°.

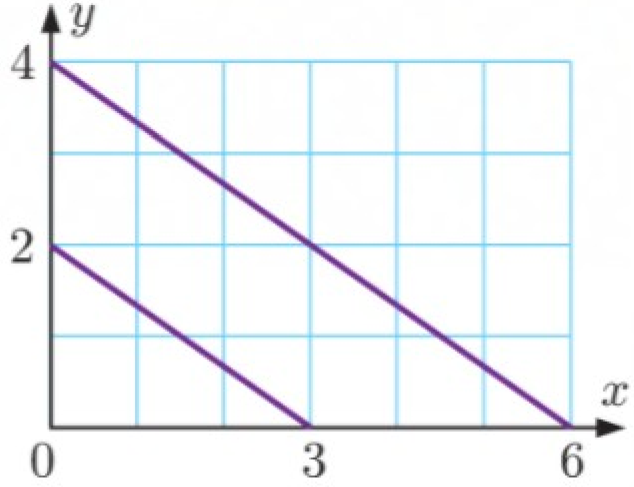
15 An observer notices an aeroplane flying directly overhead. Two minutes later the aeroplane is at an angle of elevation of 27°. Assuming the aeroplane is travelling with constant speed and altitude, what will be its angle of elevation after another two minutes?

16 A surveyor standing on a horizontal plain can see a volcano in the distance. The angle of elevation to the top of the volcano is $23°$. If the surveyor moves 750 m closer, the angle of elevation is now $37°$. Determine the height of the volcano above the plain.

17 Find the shortest distance between the two parallel lines using:

a trigonometry

b areas.



EXERCISE 12B

1 ≈ 110 m **2** $\approx 32.9°$ **3** ≈ 238 m **4** ≈ 765 m

5 ≈ 280 m **6** $6.89°$ **7** ≈ 23.5 m **8** $\approx 21.8°$

9 ≈ 15.8 cm **10** $\approx 106°$ **11** No, ≈ 0.721 cm.

12 $\approx 41.4°$ **13** $\approx 53.2°$ **14** **a** ≈ 248 m **b** ≈ 128 m

15 $\approx 14.3°$ **16** ≈ 729 m

17 **a** ≈ 1.66 units **b** ≈ 1.66 units